Lecture 10: Filter Types & Filter Design

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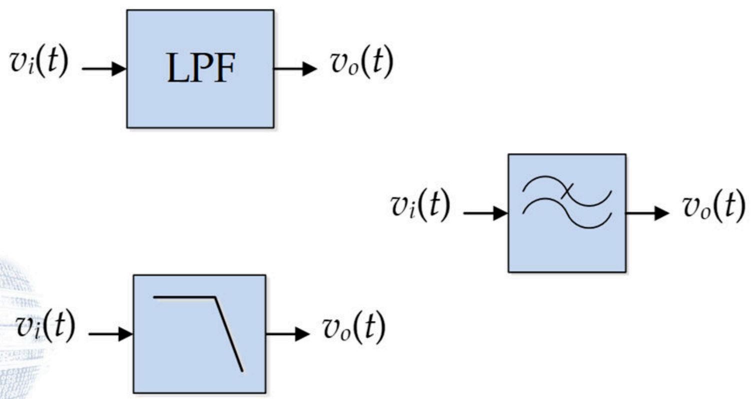
EE423: Communication Electronics

Filter Types

- Four main filter types:
  - Low-Pass Filter (LPF).
  - High-Pass Filter (HPF).
  - Band-Pass Filter (BPF).
  - Band-Stop Filter (BSF) / Notch Filter.
- Each one of these filters can be:
  - Passive Filter (uses passive components).
  - Active Filter (Amplifier allows gain > 1).

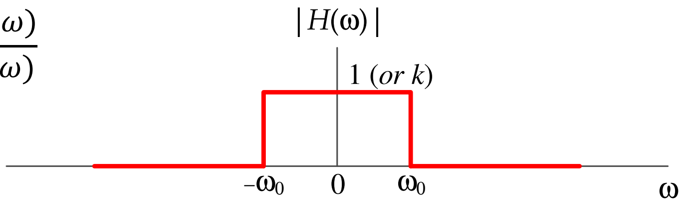
Low-Pass Filter (LPF)

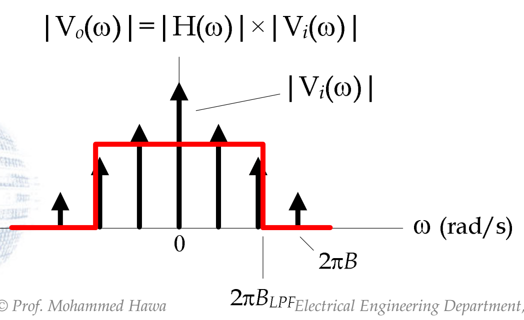
- Symbol:



Frequency Response Function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$



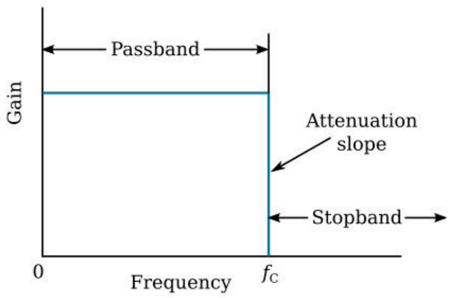


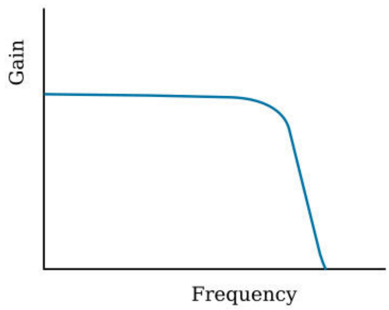
Characteristics/Specifications

- Always *centered* at 0 rad/s.
- *Bandwidth* =
  Cutoff frequency =
  Corner frequency =
  -3 dB frequency =
  ω_0 rad/s
- *Gain* = k.

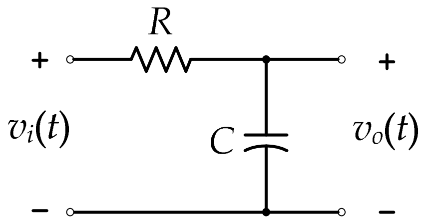
w/ Bandwidth = 5 kHz
& Gain = 2

Ideal vs. Practical Filters





Example: Passive First-Order LPF



$$Gain = 1$$

$$\omega_0 = \frac{1}{RC} \text{ rad/s}$$

$$B_{LPF} = f_{-3\text{ dB}} = f_0 = \frac{1}{2\pi RC} \text{ Hz}$$

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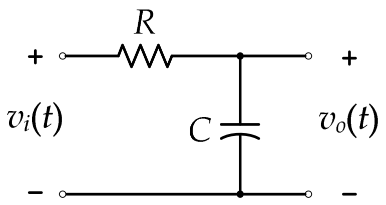
Analysis: First-Order RC Circuit



$$\boldsymbol{H}(\omega) = \frac{\boldsymbol{V_o}(\omega)}{\boldsymbol{V_i}(\omega)}$$

$$\boldsymbol{V_o}(\omega) = \frac{\boldsymbol{Z_C}(\omega)}{\boldsymbol{Z_R}(\omega) + \boldsymbol{Z_C}(\omega)}\boldsymbol{V_i}(\omega)$$

$$\frac{\boldsymbol{V_o}(\omega)}{\boldsymbol{V_i}(\omega)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

- Neglect input impedance.
- Neglect output impedance.
- Use voltage divider.
- Common form:

$$\boldsymbol{H}(s) = \frac{1}{1 + s\,RC}$$

$$\boldsymbol{H}(\omega) = \frac{1}{1 + j\omega RC}$$

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Normalization

$$H(\omega) = \frac{1}{1 + j\omega RC} = \frac{1}{1 + \frac{j\omega}{\left(\frac{1}{RC}\right)}} = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

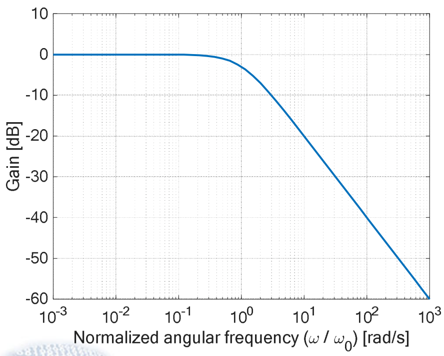
$$H(s) = \frac{1}{RC\ s + 1} = \frac{1}{\frac{s}{\left(\frac{1}{RC}\right)} + 1} = \frac{1}{\frac{s}{\omega_0} + 1}$$

$$H(s) = \frac{1}{(\tilde{s} + 1)}$$

$$H(s) = \frac{1}{(s + 1)}$$

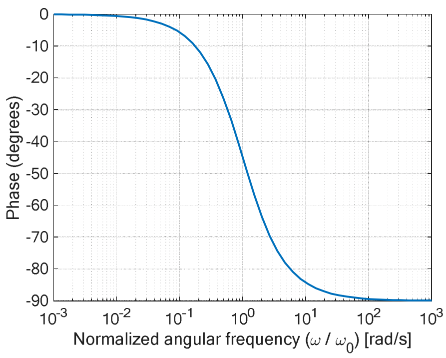
Assuming $\omega_0 = 1$ rad/s, or anything while using normalized $\tilde{s}$ (i.e., $\frac{\omega}{\omega_0}$ instead of ω).

Frequency Response Function

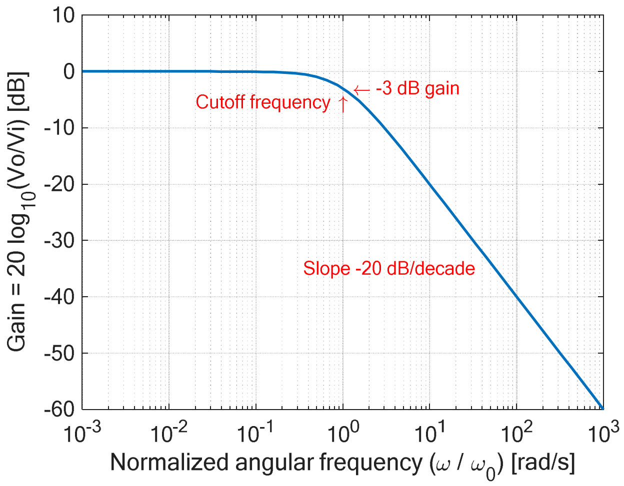


$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}} \quad ; \quad \omega_0 = \frac{1}{RC}$$

$$\angle H(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$



Magnitude Response



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Reminder: Using dB

$$Gain = \frac{Output\ Power}{Input\ Power} = \frac{P_o}{P_i} \quad (unitless)$$

$$Gain = 10 \times \log_{10}\left(\frac{P_o}{P_i}\right) \quad (dB)$$

$$\frac{P_2}{P_1}[dB] = 10 \times \log_{10}\left(\frac{P_2}{P_1}[unitless]\right) = 10 \times \log_{10}\left(\left(\frac{V_2}{V_1}\right)^2 [unitless]\right)$$

$$\frac{V_2}{V_1}[dB] = 20 \times \log_{10}\left(\frac{V_2}{V_1}[unitless]\right)$$

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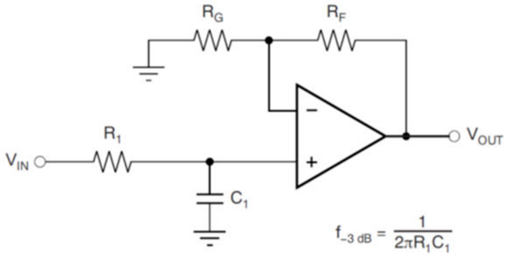
Example: Active First-Order LPF



- Active Filter = Two stages = Passive Filter followed by Amplifier

$$\boldsymbol{H}(s) = \left(1 + \frac{R_F}{R_G}\right)\left(\frac{1}{1 + s\,R_1 C_1}\right)$$

$$B_{LPF} = f_{-3\text{ dB}} = f_0 = \frac{1}{2\pi R_1 C_1}$$

$$Gain = 1 + \frac{R_F}{R_G}$$

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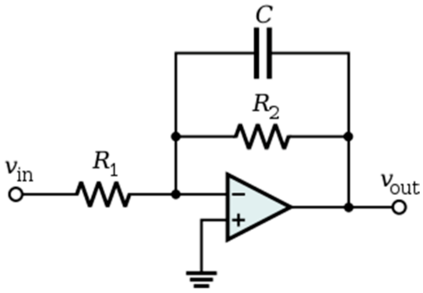
Another Example Active LPF



- Active Filter = Passive R and C components *with* Amplifier

$$\boldsymbol{H}(s) = \left(\frac{-R_2}{R_1}\right)\left(\frac{1}{1 + s\,R_2 C}\right)$$

$$B_{LPF} = f_{-3\text{ dB}} = \frac{1}{2\pi R_2 C}$$

$$Gain = \frac{-R_2}{R_1}$$

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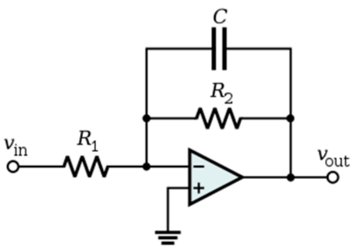
Analysis: LPF



- Neglect small input current to Op-Amp terminals.
- Voltage difference between Op-Amp input terminals is small, assumed to be zero.
- Use KCL.

$$\frac{\boldsymbol{V_o}(\omega)-0}{\boldsymbol{Z_{eq}}(\omega)}+\frac{\boldsymbol{V_i}(\omega)-0}{\boldsymbol{Z_{R_1}}(\omega)}=0$$

$$\boldsymbol{Z_{R_1}}(\omega)=R_1$$

$$\boldsymbol{Z_{eq}}(\omega)=\boldsymbol{Z_{R_2}}(\omega)||\boldsymbol{Z_C}(\omega)$$

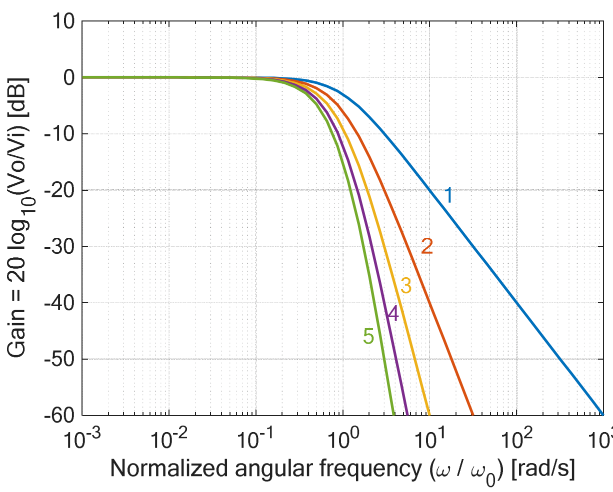
$$\boldsymbol{Z_{eq}}(\omega)=\frac{R_2\times\frac{1}{j\omega C}}{R_2+\frac{1}{j\omega C}}=\frac{R_2}{1+j\omega R_2 C}$$

$$\frac{\boldsymbol{V_o}(\omega)}{\frac{R_2}{1+j\omega R_2 C}}=-\frac{\boldsymbol{V_i}(\omega)}{R_1}$$

$$\boldsymbol{H}(\omega)=\frac{\boldsymbol{V_o}(\omega)}{\boldsymbol{V_i}(\omega)}=\frac{\frac{R_2}{1+j\omega R_2 C}}{-R_1}$$

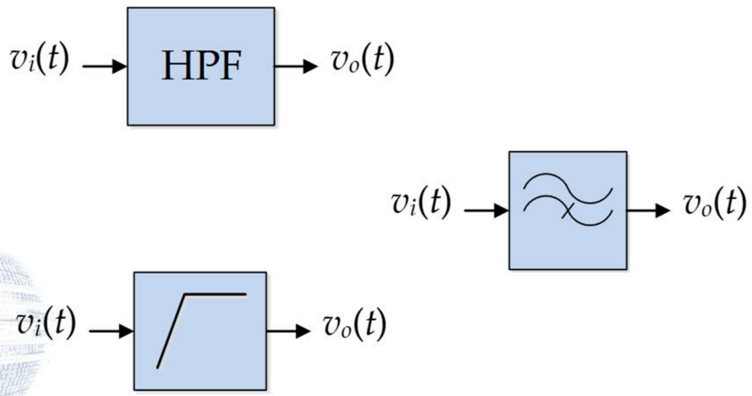
$$\boldsymbol{H}(\omega)=\left(\frac{-R_2}{R_1}\right)\left(\frac{1}{1+j\omega R_2 C}\right)$$

Improving Response (See Ideal)!



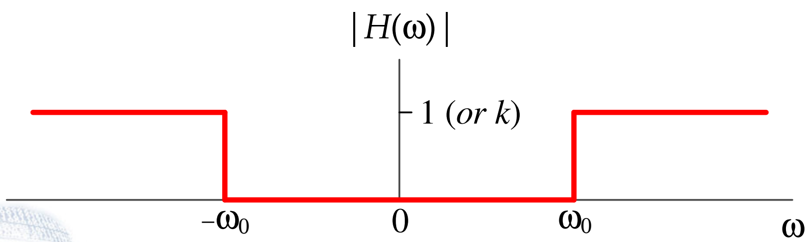
High-Pass Filter (HPF)

- Symbol:



Frequency Response Function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$



Characteristics

- Cutoff frequency = Corner frequency = ω_0 rad/s.
- Gain = k.
- No *bandwidth* defined

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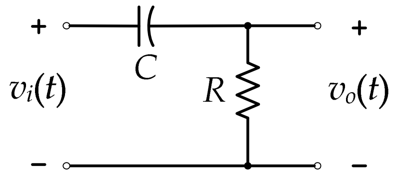
Example Circuit: First-order HPF



$$Gain = 1$$

Homework

$$\boldsymbol{H}(s) = \frac{s\,RC}{1 + s\,RC}$$

$$\omega_0 = \frac{1}{RC} \text{ rad/s}$$

$$f_{-3\text{ dB}} = f_0 = \frac{1}{2\pi RC} \text{ Hz}$$

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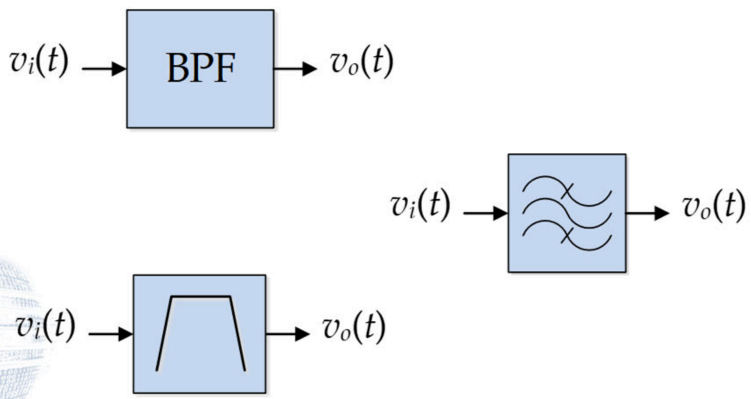
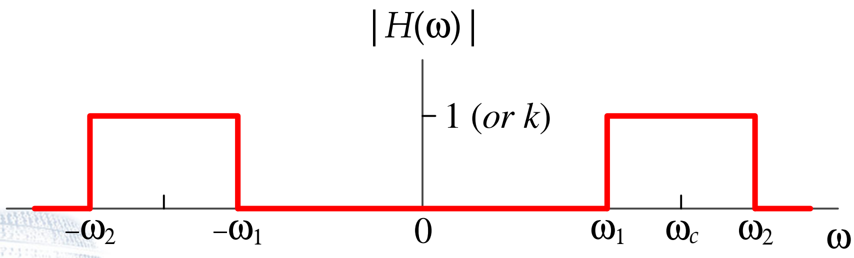
Band-Pass Filter (BPF)

- Symbol:



Frequency Response Function

$$\boldsymbol{H}(\omega) = \frac{\boldsymbol{V_o}(\omega)}{\boldsymbol{V_i}(\omega)}$$



Characteristics/Specifications

- Centered around center frequency ω_c rad/s.
- Bandwidth of filter = ω_2 - ω_1 rad/s
- Gain = k.

w/ Bandwidth = 80 kHz
Center Frequency = 100 MHz
& Gain = 1

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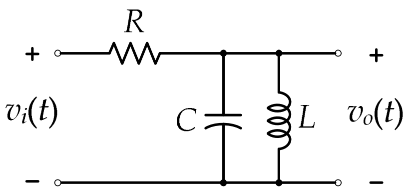
Example Circuit: LC Tank



$$f_c = f_{res} = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

$$B_{BPF} = \Delta f = \frac{R}{2\pi L} \text{ Hz}$$

$$Gain = 1$$

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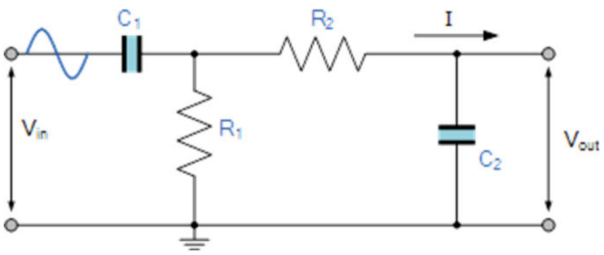
Cascading: HPF & LPF *is* BPF

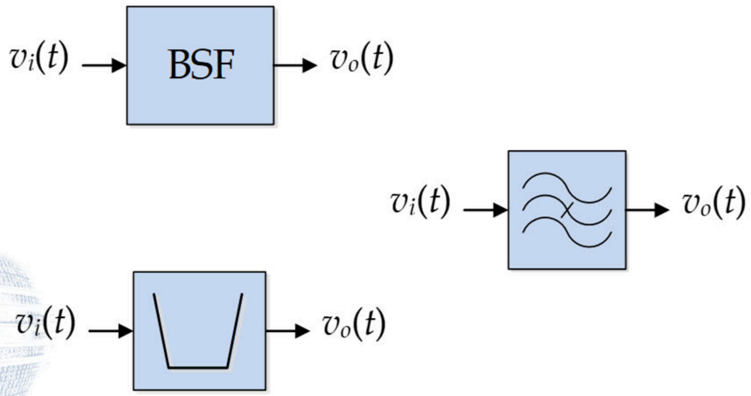
$$\boldsymbol{H}(s) = \frac{\boldsymbol{V_o}}{\boldsymbol{V_i}} = \frac{\boldsymbol{V_o}}{\boldsymbol{V_x}} \times \frac{\boldsymbol{V_x}}{\boldsymbol{V_i}} \qquad \boldsymbol{H}(s) = \boldsymbol{H_1}(s) \times \boldsymbol{H_2}(s)$$



$$\omega_{0,\mathrm{LPF}} > \omega_{0,\mathrm{HPF}}$$

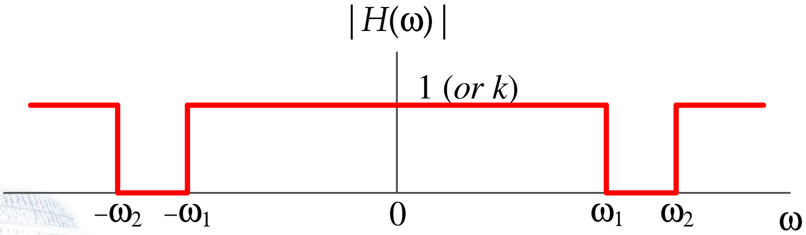
Band-Stop Filter

- Symbol:

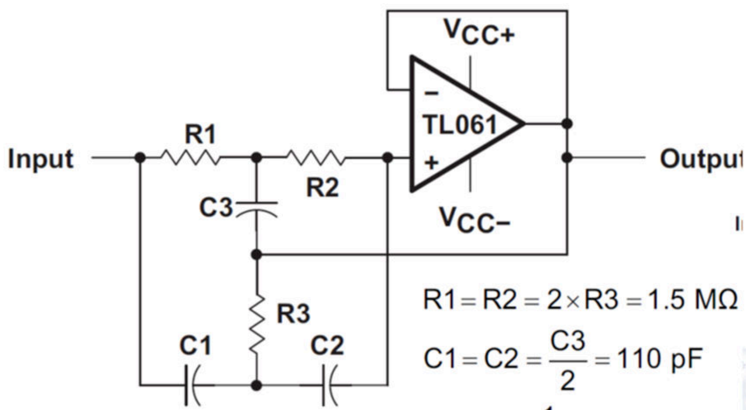


Frequency Response Function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$



Example: Twin T Active Notch Filter

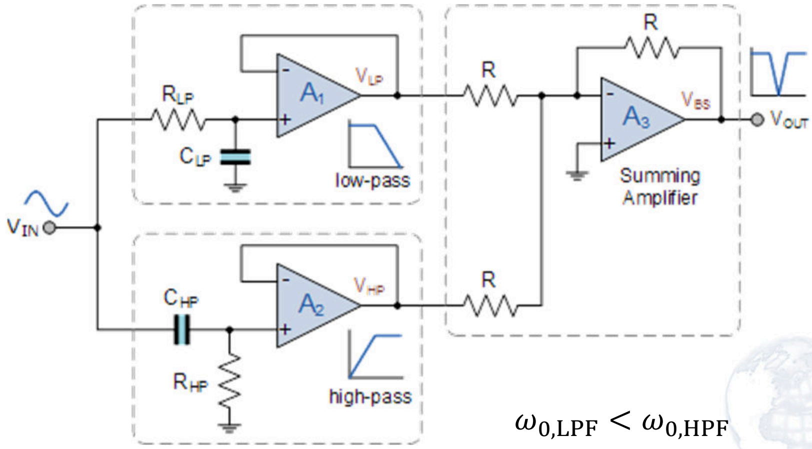


$R1 = R2 = 2 \times R3 = 1.5\ M\Omega$

$C1 = C2 = \frac{C3}{2} = 110\ pF$

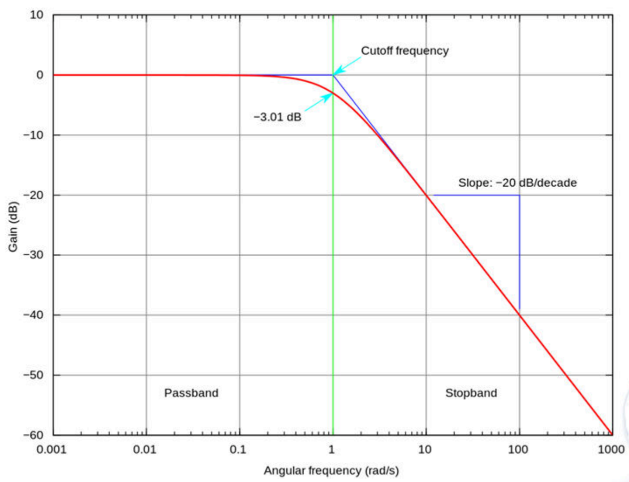
$f_o = \frac{1}{2\pi \times R1 \times C1} = 1\ kHz$

Another Notch: Add *not* Cascade

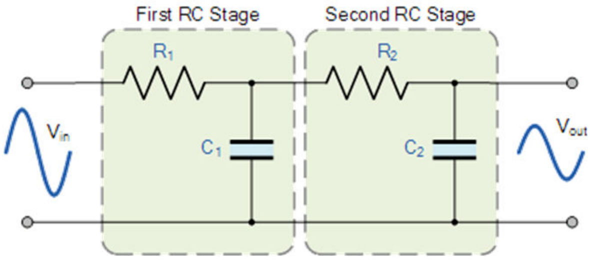


$\omega_{0,\text{LPF}} < \omega_{0,\text{HPF}}$

Controlling Order of Filter



Multi-Stage Design: *Ladder Network*



$$\boldsymbol{H}(s) = \left(\frac{1}{1 + s\,R_1C_1}\right)\left(\frac{1}{1 + s\,R_2C_2}\right) = \frac{1}{(1 + s\,R_1C_1)(1 + s\,R_2C_2)}$$

$$\boldsymbol{H}(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + b_1 s^1 + b_0}{a_n s^n + a_{n-1}s^{n-1} + a_1 s^1 + a_0} = \frac{B(s)}{A(s)}$$

Ladder Network: LPF Order 1 to 5

$$H_1(s) = \frac{1}{1 + s\,RC}$$

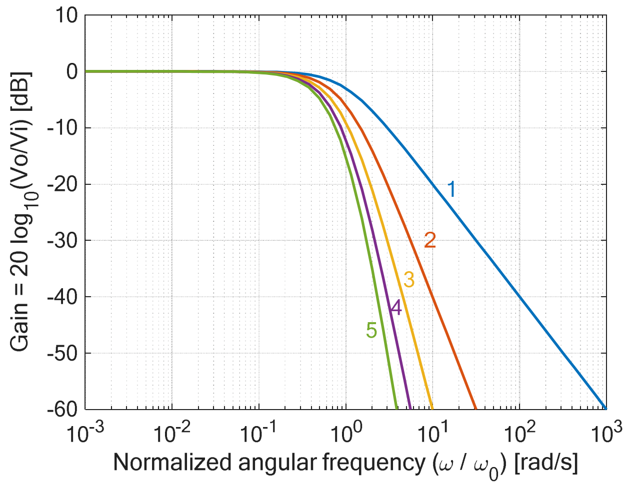
$$H_2(s) = \left(\frac{1}{1 + s\,RC}\right)\left(\frac{1}{1 + sRC}\right) = \frac{1}{(1 + sRC)(1 + sRC)}$$

$$H_3(s) = \left(\frac{1}{1 + s\,RC}\right)\left(\frac{1}{1 + sRC}\right)\left(\frac{1}{1 + sRC}\right) = \frac{1}{(1 + sRC)(1 + sRC)(1 + sRC)}$$

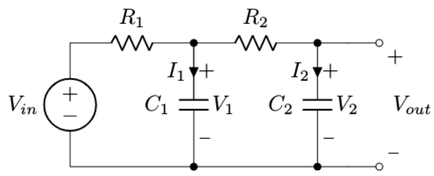
$$H_4(s) = \frac{1}{(1 + sRC)(1 + sRC)(1 + sRC)(1 + sRC)}$$

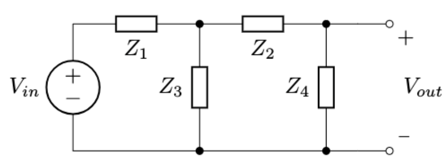
$$H_5(s) = \frac{1}{(1 + sRC)(1 + sRC)(1 + sRC)(1 + sRC)(1 + sRC)}$$

Ladder Network: LPF Order 1 to 5



Just Be More Accurate



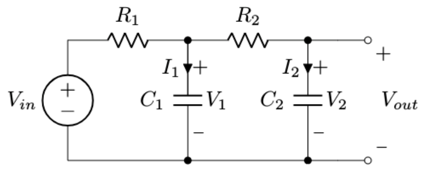


$$\frac{V_o}{V_i} = \frac{Z_4}{Z_2 + Z_4} \cdot \frac{Z_{eq}}{Z_{eq} + Z_1}$$

$$Z_{eq}(\omega) = Z_3 || (Z_2 + Z_4)$$

$$\frac{V_o}{V_i} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_1 Z_3 + Z_1 Z_4 + Z_2 Z_3 + Z_3 Z_4}$$

Accurate Equation *Or* Employ *Buffer*

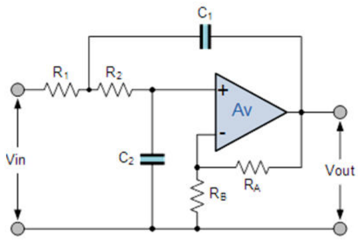


$$\frac{\boldsymbol{V_o}}{\boldsymbol{V_i}} = \frac{\boldsymbol{Z_3 Z_4}}{\boldsymbol{Z_1 Z_2 + Z_1 Z_3 + Z_1 Z_4 + Z_2 Z_3 + Z_3 Z_4}}$$

$$\boldsymbol{Z_1} = R_1, \boldsymbol{Z_2} = R_2, \boldsymbol{Z_3} = \frac{1}{sC_1}, \boldsymbol{Z_4} = \frac{1}{sC_2}$$

$$\boldsymbol{H}(s) = \frac{\boldsymbol{V_o}}{\boldsymbol{V_i}} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2)s + 1}$$

Second Order Active Filter *(Sallen–Key Topology)*



Gain (Av) = $1 + \frac{R_A}{R_B}$

If Resistor and Capacitor values are different: $f_C = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$

If Resistor and Capacitor values are the same: $f_C = \frac{1}{2\pi RC}$

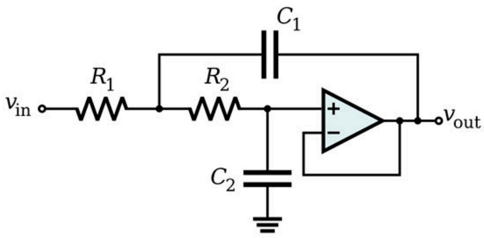
$$\boldsymbol{Z_1} = R_1, \boldsymbol{Z_2} = R_2, \boldsymbol{Z_3} = \frac{1}{sC_1}, \boldsymbol{Z_4} = \frac{1}{sC_2}$$

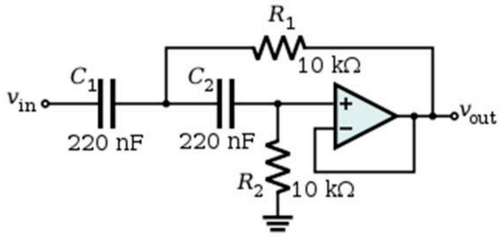
$$K = \left(1 + \frac{R_A}{R_B}\right)$$

Homework

$$\boldsymbol{H}(s) = \frac{\boldsymbol{V_o}}{\boldsymbol{V_i}} = \frac{K}{(R_1 R_2 C_1 C_2)\, s^2 + \left(R_1 C_2 + R_2 C_2 + \left(R_1 C_1 (1 - K)\right)\right)\, s + 1}$$

Notice Sallen–Key Topology Variations (*also* LPF vs. HPF)





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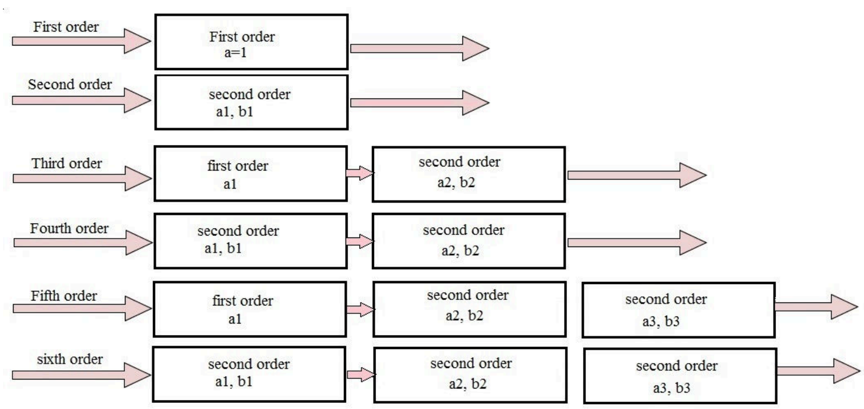
Blocks: Get Desired Poles & Zeros



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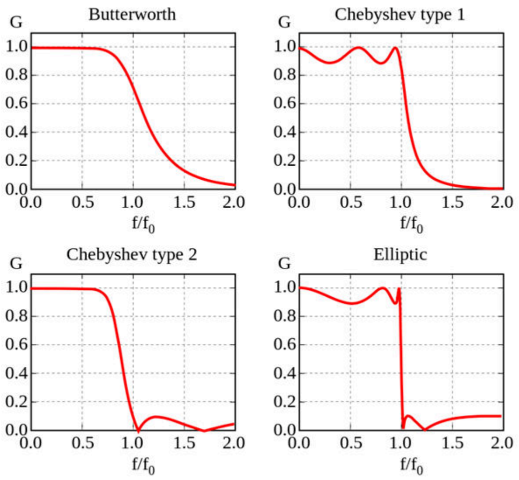
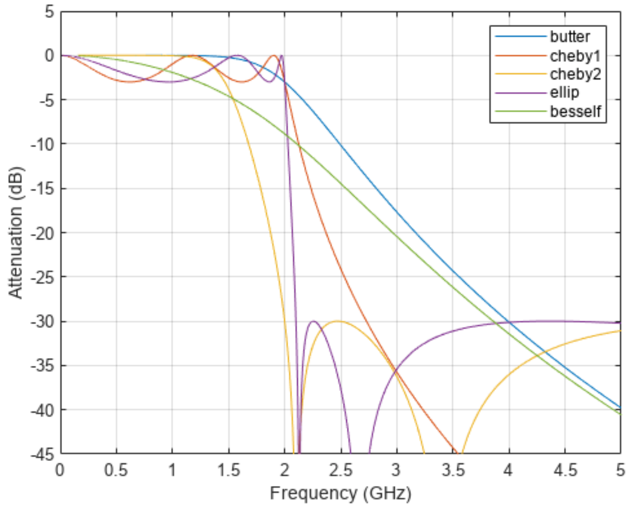
Practical Filter Response Shapes

- **Butterworth** filter: maximally flat amplitude response.
- **Chebyshev Type I** filter: ripple in the passband.
- **Chebyshev Type II** filter: ripple in the stopband.
- **Elliptic** filter: much faster drop with ripple in the passband and stopband.
- **Bessel** filter: maximally flat group delay. Provides good linear phase.
- *And more …*

Example LPF Response Shapes



Comparison



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Filter Design Steps

1. Decide the requirements (specifications) desired for the filter.
2. Find the transfer function that satisfies these requirements (i.e., find $B(s)$ and $A(s)$)

$$\boldsymbol{H}(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + b_1 s^1 + b_0}{a_n s^n + a_{n-1} s^{n-1} + a_1 s^1 + a_0} = \frac{B(s)}{A(s)}$$

3. Find the circuit or multi-stage circuits that have this transfer function.

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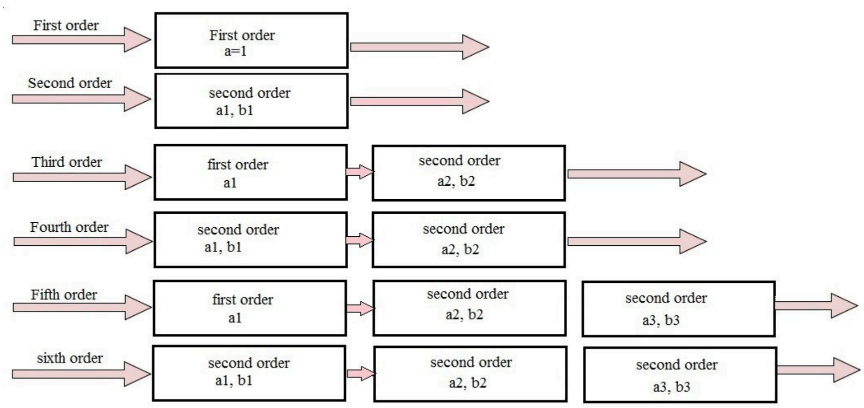
Cascading: Based on Poles & Zeros



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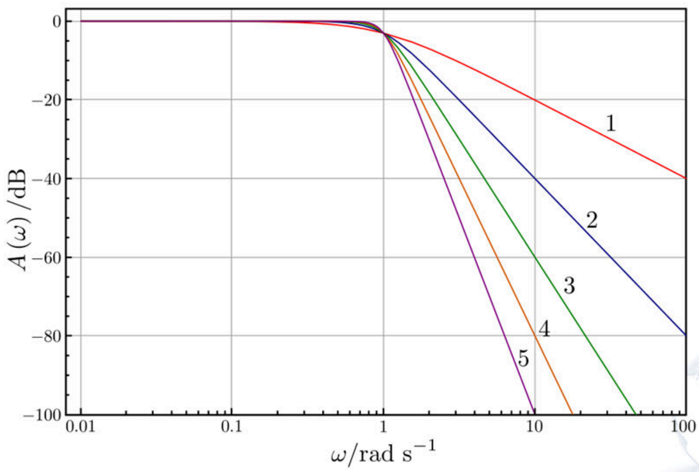
Butterworth LPF order 1 to 5



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Butterworth $A_n(s)$

n	Butterworth Polynomials $A_n(s)$
1	$(s+1)$
2	$\left(s^2+\sqrt{2}\,s+1\right)$
3	$(s+1)(s^2+s+1)=s^3+2s^2+2s+1$
4	$\left(s^2+\sqrt{2-\sqrt{2}}\,s+1\right)\left(s^2+\sqrt{2+\sqrt{2}}\,s+1\right)$
5	$(s+1)\left(s^2+\frac{2}{1+\sqrt{5}}s+1\right)\left(s^2+\frac{1+\sqrt{5}}{2}s+1\right)$
6	$\left(s^2+\sqrt{2-\sqrt{3}}\,s+1\right)\left(s^2+\sqrt{2}\,s+1\right)\left(s^2+\sqrt{2+\sqrt{3}}\,s+1\right)$

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Or $A_n(s)=\sum_{k=0}^{n} a_k s^k$ Coefficients

Butterworth Coefficients a_k to Four Decimal Places

n	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
1	1	1									
2	1	1.4142	1								
3	1	2	2	1							
4	1	2.6131	3.4142	2.6131	1						
5	1	3.2361	5.2361	5.2361	3.2361	1					
6	1	3.8637	7.4641	9.1416	7.4641	3.8637	1				
7	1	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940	1			
8	1	5.1258	13.1371	21.8462	25.6884	21.8462	13.1371	5.1258	1		
9	1	5.7588	16.5817	31.1634	41.9864	41.9864	31.1634	16.5817	5.7588	1	
10	1	6.3925	20.4317	42.8021	64.8824	74.2334	64.8824	42.8021	20.4317	6.3925	1

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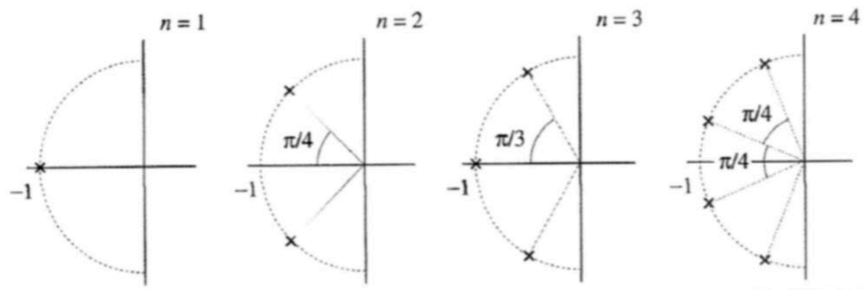
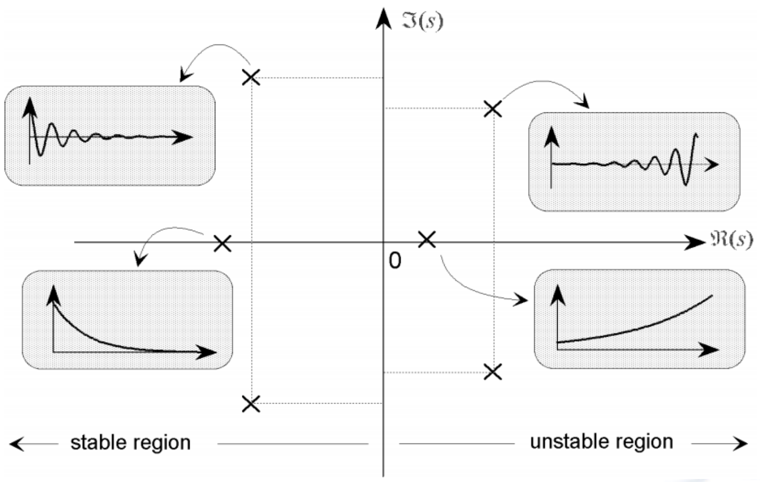
Poles on s-plane (pole–zero plot) for Butterworth Order 1 to 4



For stability, we only use poles with negative real parts (left half of s-plane)



Example: LPF Design

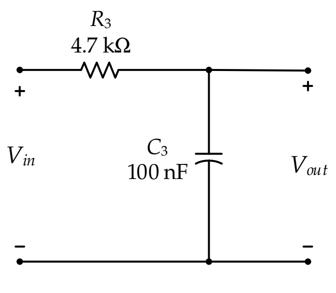
Q. Design a third-order Butterworth LPF with B = 340 Hz

$$\boldsymbol{H}(s) = \frac{1}{s^3 + 2s^2 + 2s + 1} = \frac{1}{(s^2 + s + 1)(s + 1)}$$

$$\boldsymbol{H}(s) = \frac{1}{(\tilde{s}^2 + \tilde{s} + 1)} \cdot \frac{1}{(\tilde{s} + 1)}$$

$$\boldsymbol{H}(s) = \frac{1}{\left(\left(\frac{s}{\omega_0}\right)^2 + \left(\frac{s}{\omega_0}\right) + 1\right)} \cdot \frac{1}{\left(\left(\frac{s}{\omega_0}\right) + 1\right)}$$

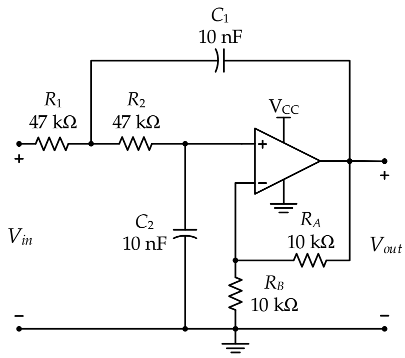
Second Stage: First-Order Passive LPF



$$\boldsymbol{H}(s) = \frac{\boldsymbol{V_o}}{\boldsymbol{V_i}} = \frac{1}{RC\,s + 1} = \frac{1}{\frac{s}{\left(\frac{1}{RC}\right)} + 1}$$

$$f_0 = \frac{1}{2\pi RC} = 338.6275\ Hz$$

First Stage: Second-Order Active LPF



$$\boldsymbol{H}(s) = \frac{\boldsymbol{V_o}}{\boldsymbol{V_i}} = \frac{K}{(R_1R_2C_1C_2)\, s^2 + \Big(R_1C_2 + R_2C_2 + \big(R_1C_1(1-K)\big)\Big)\, s + 1}$$

$$\boldsymbol{H}(s) = \frac{\boldsymbol{V_o}}{\boldsymbol{V_i}} = \frac{K}{\left(\frac{s}{\frac{1}{RC}}\right)^2 + (3-K)\left(\frac{s}{\frac{1}{RC}}\right) + 1}$$

Calculations

$$3 - K = 1$$

$$K = 2 = \left(1 + \frac{R_A}{R_B}\right)$$

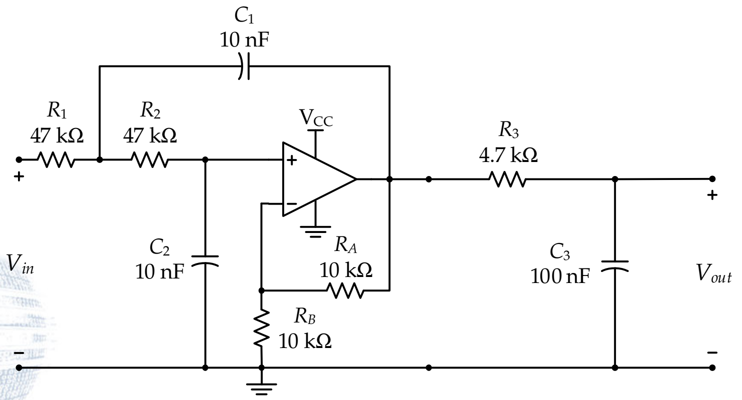
$$R_A = R_B$$

$$f_0 = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 47 \times 10^3 \times 10 \times 10^{-9}}$$

$$f_0 = 338.6275\ Hz$$

Final Solution: Two Stages

Q. Why active filter is first stage, not second stage?



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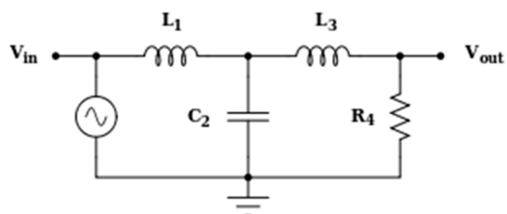
Homework

- In a ladder network (also called Cauer topology), we can use L in addition to R and C components, if needed.



- Determine the transfer function $\boldsymbol{H}(s) = \boldsymbol{V_o}/\boldsymbol{V_i}$ for the above circuit.
- Show that it is a third-order Butterworth LPF.
- Find the cutoff frequency for L_1 = 1.5 H, C_2 = 1.3333 F, L_3 = 0.5 H, and R_4 = 1 Ω. *Solution*: $\omega_0 = 1$ rad/s.

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Chebyshev Type I Filter

- The gain response $G_n(\omega)$, or the magnitude of the transfer function $H_n(s)$ evaluated at $s = j\omega$, of the nth-order Chebyshev Type I LPF is:

$$G_n(\omega) = |H_n(j\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 T_n^2(\omega/\omega_0)}}$$

- ε is the ripple factor.
- ω_0 is the cutoff frequency.
- $T_n(x)$ is the Chebyshev polynomial of the first kind and nth order.

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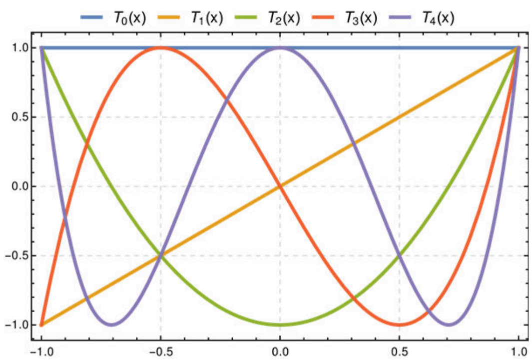
Chebyshev Polynomial

- Chebyshev polynomial of the first kind and nth order satisfies the condition $T_n(\cos\theta) = \cos(n\theta)$



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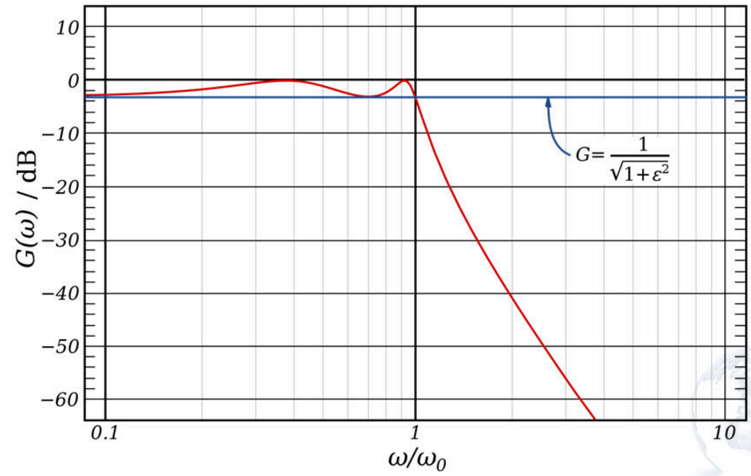
Chebyshev Type I LPF



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Chebyshev Type I Poles

- The transfer function of Chebyshev Type I is:

$$H(s) = \frac{1}{2^{n-1}\varepsilon} \prod_{m=1}^{n} \frac{1}{(s - s_{pm}^{-})}$$

- The s_{pm}^{-} poles are in the left part of the s-domain:

$$s_{pm}^{-} = -\sinh\left(\frac{1}{n}\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right)\sin\left(\frac{\pi}{2}\frac{2m-1}{n}\right) + j\cosh\left(\frac{1}{n}\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right)\cos\left(\frac{\pi}{2}\frac{2m-1}{n}\right)$$

- where $m = 1,2,\ldots,n$.

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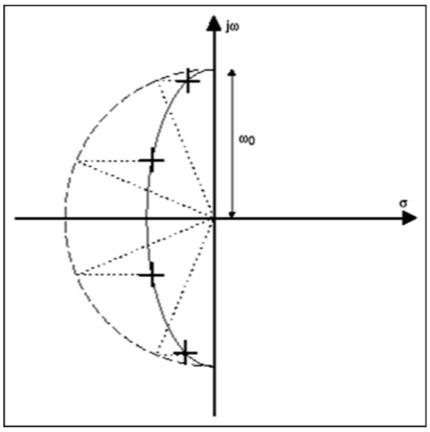
Chebyshev Type I Poles (Order 4)



MATLAB Can Calculate Poles

cheby1

Chebyshev Type I filter design

R2024a

collapse all in page

Syntax

```
[b,a] = cheby1(n,Rp,Wp)
[b,a] = cheby1(n,Rp,Wp,ftype)

[z,p,k] = cheby1(___)
[A,B,C,D] = cheby1(___)

[___] = cheby1(___,'s')
```

Description

`[b,a] = cheby1(n,Rp,Wp)` returns the transfer function coefficients of an `n`th-order lowpass digital Chebyshev Type I filter with normalized passband edge frequency `Wp` and `Rp` decibels of peak-to-peak passband ripple. example

`[b,a] = cheby1(n,Rp,Wp,ftype)` designs a lowpass, highpass, bandpass, or bandstop Chebyshev Type I filter, depending on the value of `ftype` and the number of elements of `Wp`. The resulting bandpass and bandstop designs are of order 2`n`. example

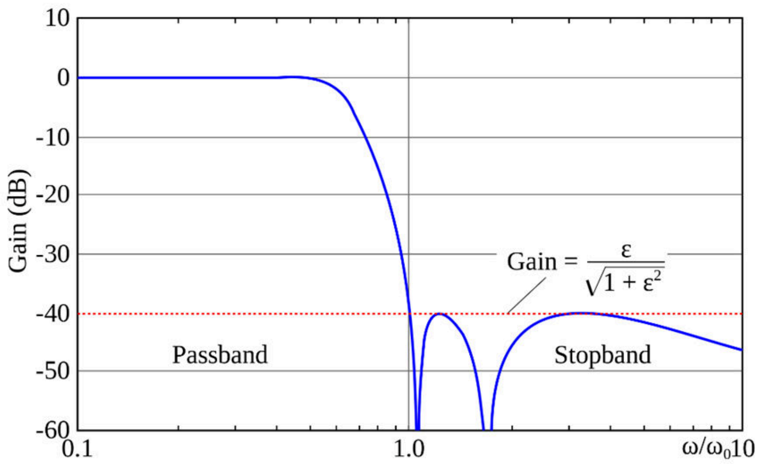
Chebyshev Type II Filter
(*Inverse Chebyshev Filter*)

$$|H_n(j\omega)| = \frac{1}{\sqrt{1 + \frac{1}{\varepsilon^2 T_n^2(\omega_0/\omega)}}} = \sqrt{\frac{\varepsilon^2 T_n^2(\omega_0/\omega)}{1 + \varepsilon^2 T_n^2(\omega_0/\omega)}}$$

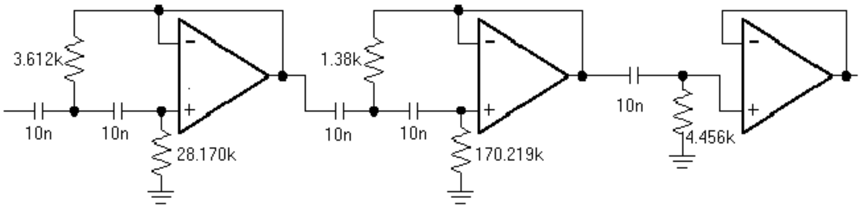
$$\frac{1}{s_{pm}^{-}} = -\sinh\left(\frac{1}{n}\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right)\sin\left(\frac{\pi}{2}\frac{2m-1}{n}\right) + j\cosh\left(\frac{1}{n}\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right)\cos\left(\frac{\pi}{2}\frac{2m-1}{n}\right)$$

- Less popular than Type I.

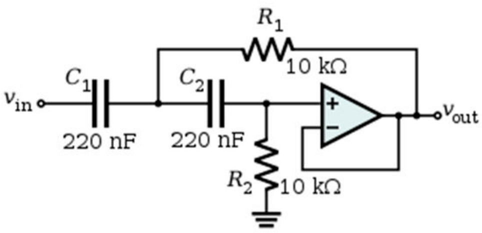
Chebyshev Type II Gain



Ex: 5th Order Chebyshev HPF



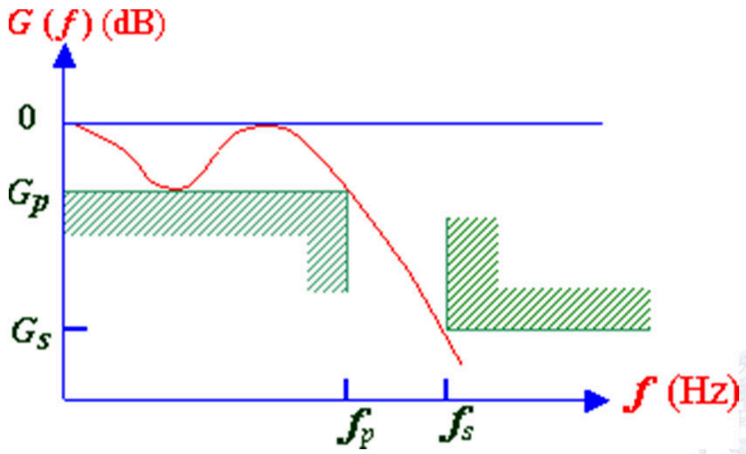
Homework: Unity-Gain Sallen–Key Active HPF



$$\boldsymbol{H}(s) = \frac{\boldsymbol{V_o}}{\boldsymbol{V_i}} = \frac{s^2}{s^2 + \left(\frac{C_1 + C_2}{R_2 C_1 C_2}\right) s + \frac{1}{R_1 R_2 C_1 C_2}}$$

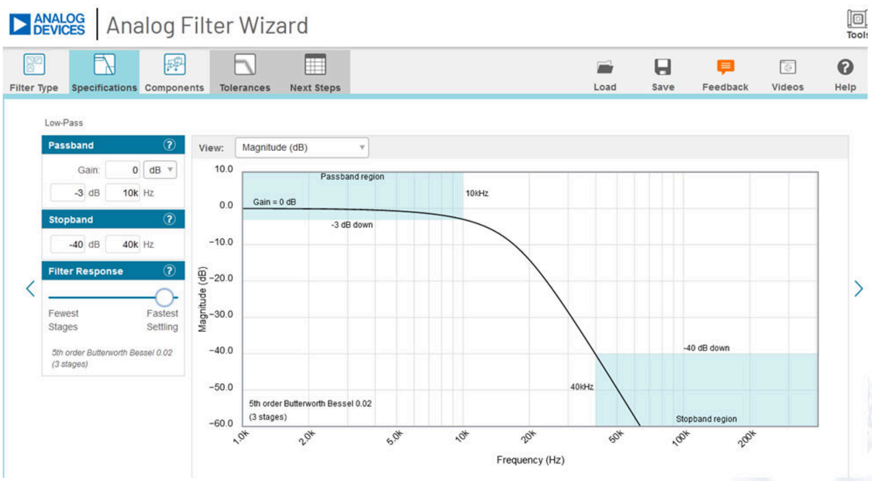
$$\boldsymbol{H}(s) = \frac{\boldsymbol{V_o}}{\boldsymbol{V_i}} = \frac{(R_1 R_2 C_1 C_2)\, s^2}{(R_1 R_2 C_1 C_2)\, s^2 + (R_1 C_1 + R_1 C_2)\, s + 1}$$

Filter Specification Diagram



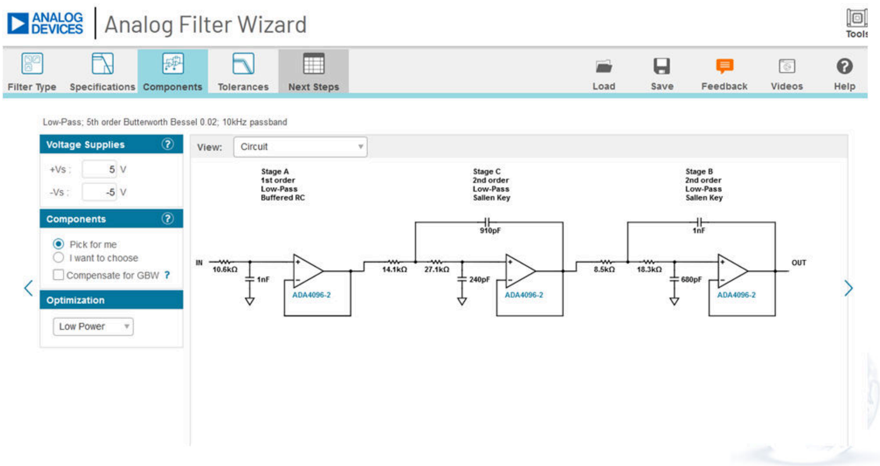
Analog Devices Filter Design Tool

https://tools.analog.com/en/filterwizard/

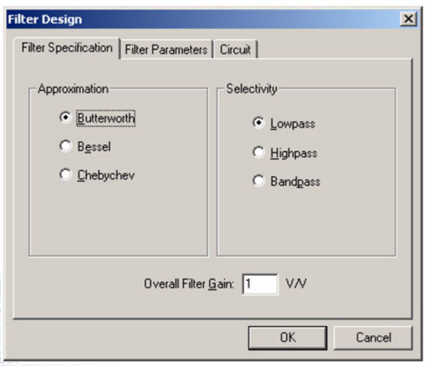


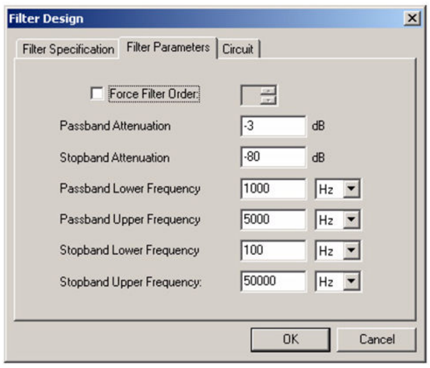
Analog Devices Filter Design Tool

https://tools.analog.com/en/filterwizard/

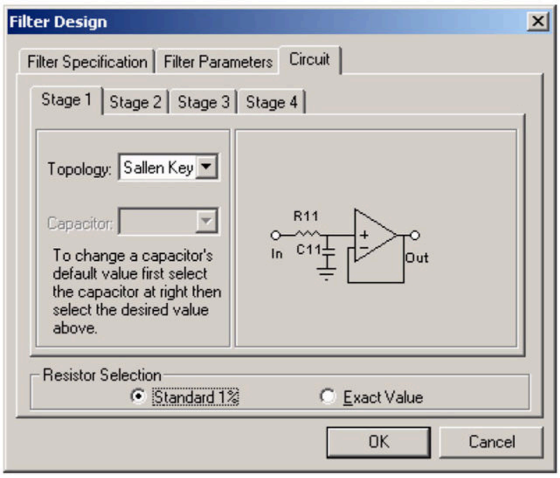


Microchip FilterLab





Microchip FilterLab



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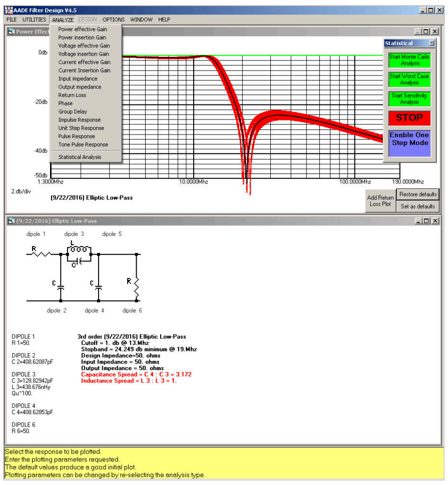
AADE Filter Design



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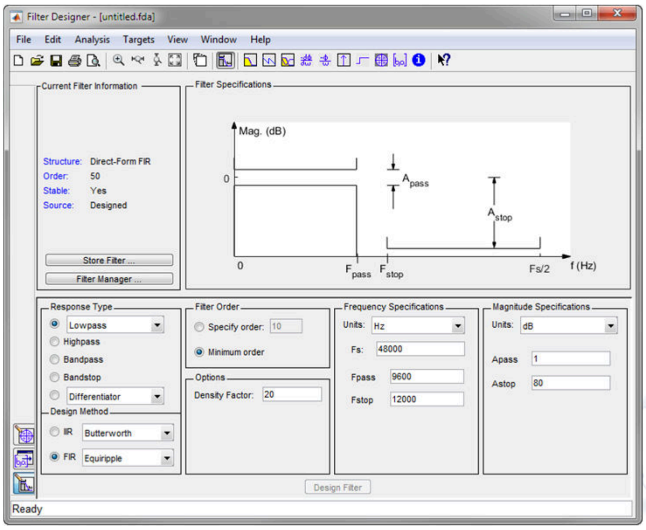
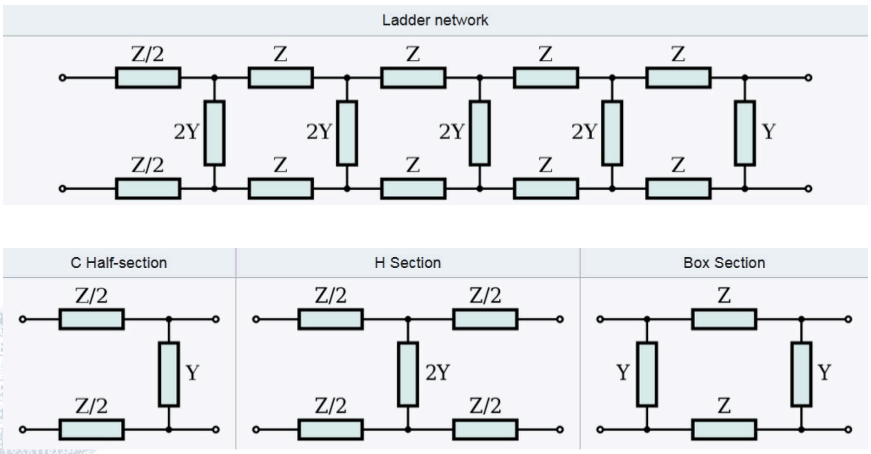
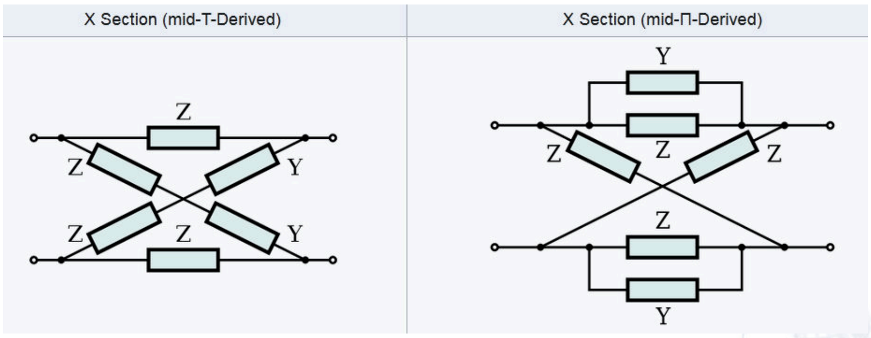
MATLAB Filter Designer



Many Filter Topologies

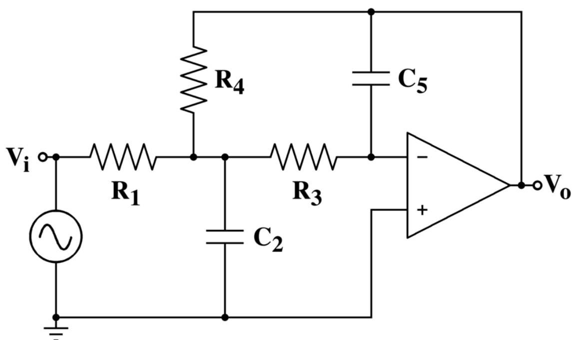


Passive …



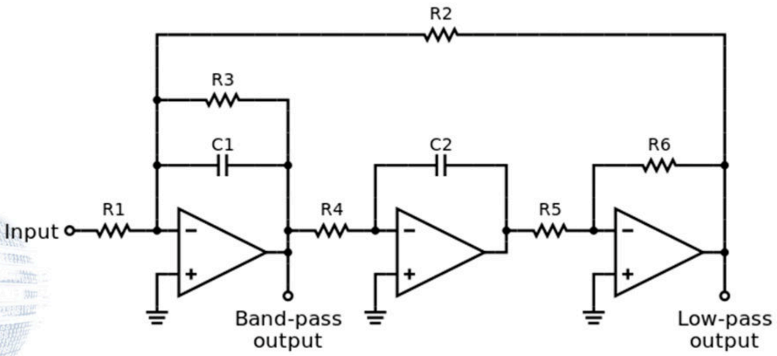
… *and* Active

- Multiple feedback topology (MFB) (active filter).



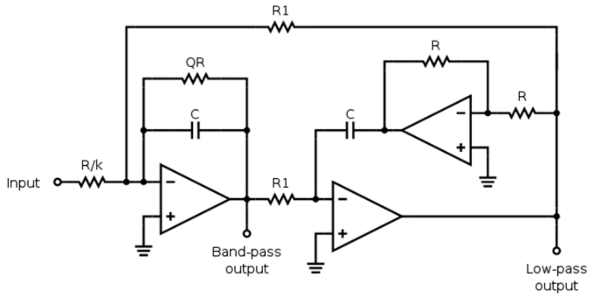
ICs Allow More Op-Amps

- Common Tow-Thomas **biquad** filter topology.



Filters in ICs

- Akerberg-Mossberg **biquad** filter topology.



- Switched-capacitor IC-based analog filters.
- Mechanical filters, etc.

DSP: FIR (finite impulse response)

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_M z^{-M}}$$